MODELLING MEAN SURFACE TEMPERATURE OF NIGERIA USING GEOSTATISTICAL APPROACH

By

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DEPARTMENT OF STATISTICS
FACULTY OF PHYSICAL SCIENCES
AHMADU BELLO UNIVERSITY, ZARIA

JANUARY, 2018.
DECLARATION

I declare that the work in this Dissertation entitled Modelling Mean Surface Temperature of Nigeria using Geostatistical Approach has been carried out by me in the Department of Statistics, Faculty of Physical Sciences. The information derived from the literature has been duly acknowledge in the text and in a list of references provided. No part of this Dissertation was previously presented for another degree or diploma in this or another institution.

Name of Student ____________________ Sign ____________________ Date ________________
CERTIFICATION

This Dissertation entitled MODELING MEAN SURFACE TEMPERATURE OF NIGERIA USING GEOSTATISTICAL APPROACH by Attah Yakubu GARBA meets the regulations governing the award of the degree of Master of Science (MSc) Statistics of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

This work is dedicated to my beloved Father: Late Garba Attah Avuzhen
ACKNOWLEDGEMENTS

I want to thank God Almighty for his mercies, provision and protection over my life generally and specifically for the experiences acquired and time well spent at Ahmadu Bello University (A.B.U), Zaria. Collectively, I sincerely appreciate the efforts of all the staff in the department of statistics and mathematics for making it a reality in my life. To mention but a few: Professor A. Isah and Dr. A. Yahaya my supervisors, Professor Gulumbe, Professor O.E Asiribo, Professor Junaidu of Computer Science Department, Mr. Reuben and Dr. A. J. Alkali in Mathematics Department. Equally, I want to say thank you to the following friends for their assistance in one way or the other towards the success of my study: Mallam Abdullahi Usman of Department of Statistic, F.U.T. Minna, Mr Leo and Efiki of NBS, Central area, Garki, Chief Abu, Aren Eggon Moa Zaria, Ewuga, Moses, Fatima, Synthia, Oga Farook of UEBEB area 2, Debora, Ruth, Kele, Ahmed Claimist, Timothy, Murni, Abims, Chief Agada and Yohanna. Finally, I want to thank the Federal Capital Territory Administration’s Education Authority: Universal Basic Education Board (UBEB) Management for the opportunity granted to me, all the staff of Junior Day Secondary School Kwali especially Mr Cooler and my entire family especially my Mummy, Benita, Magdalene, Success and Otsechahe, to mention but a few for standing beside me to achieve this greatness.
ABSTRACT

Understanding spatial variability of mean surface temperature (MST) of Nigeria is necessary for ecological restoration and national planning toward effects of unstable climate conditions. This study developed models for MST derived from two geostatistical procedures and Multiple Linear Regression (MLR) using measurements of monthly MST in Nigeria. The two geostatistical approaches were Ordinary Kriging (OK) and Regression Kriging (RK). The Ordinary kriging (OK) was developed in two dimensions with isotropy and in three dimensional plane with anisotropy and regression kriging (RK) that employed both correlation with independent variables and spatial autocorrelation simultaneously. Four statistics were considered to evaluate the performance of the approaches used. For the fitted MLR model, some of the predictors were significant at p-value of 0.05 with $R^2$ equal to 88 percent. The one-leave-out cross-validation indicated that RK produced minimum errors compared to OK model. The OK with zonal anisotropic shown that the spatial continuity in the directions of North and North East were stronger than the directions of East and South East. The kriging weights for OK and RK were similar as shown in the maps. The RK model outperformed the OK and the MLR and was therefore recommended for Long term prediction of mean Surface Temperature of Nigeria.
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CHAPTER ONE

GENERAL INTRODUCTION

1.1 Background of the study

When we speak of temperature of an object, we often associate this concept with how hot or cool the object feels. Scientifically, temperature is a property of an object that decides the direction of flow of energy of the object when being encountered with another object. Temperature measurements were dated back to 1600 by Galileo and Santario. Different disciplines have different point of interest on the concept temperature, thus, thermometer (measuring instrument) varies, depending on the size, nature and the position of the body on the earth. That is, it could be human body temperature, soil temperature, temperature at a particular depth in an ocean, temperature at a particular altitude above the sea level or on the surface of the earth. The instrument thermometer is used to measure temperature and it ranges from clinical to meteorological thermometers. Surface temperature therefore is the measurement of temperature considered on the earth surface and it is the focus of this research.

Surface temperature is one of the input variables for land evaluation and characterisation systems, as well as hydrological and ecological models. These models use surface temperature to derive processes such as evapotranspiration, soil decomposition, and plant productivity. Hence, forest management and research need this climatic variable as a basis for understanding many processes, as it is the main factor affecting vegetation distribution, in the sense that its action is felt over wide areas of the earth’s surface (Dodson and Marks, 1997).

Surface temperature is an important site characteristic used in determining site suitability for agricultural and forest crops (Hudson and Wackernagel, 1994), and it is used in parameterizing the habitat of plant species (Rubio et al., 2002; Sanchez-
Palomares et al., 2003) and in determining the patterns of vegetation zonation (Richardson et al., 2004). Moreover, surface temperature is a factor related to plant productivity; as it is connected with the length of the vegetative period and evapotranspiration. Different indexes have been developed for relationships between components of climate and quantitative variables of vegetation communities. These indexes are known as potential productivity indexes and they are reviewed in Hagglund (1981) and Del Valle (2004). Temperature can also be considered as a limitation factor, and many studies have been carried out with temperature as a threshold for different processes, for instance, the minimum temperature which may lead to lethal damage of tissue or whole tree seedlings under different situations of stand density, topography or soil preparation (Blennow, 1993, 1998).

Geostatistics originated from the mining and petroleum industries in 1950’s by Danie Krige and was further developed by Georges Matheron in 1960’s. In both industries, geostatistics was successfully applied to solve problems where decision concerning expensive operation were based on interpretation from sparse data located in space. It has since been related to many other earth science related fields such as hydrogeology, hydrology, soil science and forestry. According to (Goovaerts, 1997) geostatistics is a subset of statistics specialized in analysis and interpretation of geographically referenced data. (Olea, 1999) considered geostatistics to be the only scientific field specialized in the analysis of spatial data. Isah and Usman (2015) stated that geostatistics is a branch of statistics that specializes in the analysis and interpretation of any spatially (temporally) referenced data.

Therefore, spatial modelling of climate variables, such as surface temperature, is of interest for forestry science, ecology and meteorology. Consequently, many different methods have been developed to generate regional maps from point data, based on the continuity of temperature and its strong dependency on elevation; on a global average, temperature decreases around $0.658^\circ C/100$ m in altitude (Barry and Chorley, 1987, although this rate may vary with the season and the geographic
situation (Goodale et al., 1998), and in relation to diurnal effects (Richardson et al., 2004). According to Hengl (2009), the top 10 application fields of geostatistics are: Geosciences, Water resources, Environmental sciences, Soil science, Ecology, Mathematics and Statistics, Civil Engineering, Petroleum Engineering and Meteorology. Central to Fernado et al., (2007), Isah (2013) and Goovaerts (1999), geostatistics is the best analytical tool for spatial referenced data analysis. Thus, we intend to model the mean surface temperature of Nigeria and bring forth appropriate methodology for long term prediction of mean surface temperature of Nigeria, a geostatistical approach as widely recommended tool for spatial data analysis. As recommended by (Olusina and Odumade, (2012), and Amadi et al., 2014} and other researchers on climate change, doing our research in 2016 - 2017 is as good as filling the gap.

1.2 Statement of the Problem.

National Aeronautics and Space Administration (NASA) Goddard Institute for Space Studies Scientists (2016) reported an upward trend in temperature of the world, of which Nigeria is not excluded. According to NASA, the average temperature for 2006 and 2009 tie with 2013, being the seventh warmest years since 1880. NASA further emphasized that the weather pattern will cause drastic change in climate which resultant effect could bring about poor production and extinction of plants and animals. From the works reviewed, much is yet to be done on temperature research using geostatistical approach and a developing country like Nigeria needs to have a geostatistical model (Regression Kriging Model) that is capable of predicting future mean surface temperature values with higher degree of accuracy. Regression Kriging is a hybrid model [combination of regression model and kriging model]. Hengl (2009) combined simple linear regression and ordinary kriging. Fernando et al., (2007) used ordinary Kriging with external drift etc. In this research, we are combining Multiple Linear Regression with Ordinary Kriging.
1.3 Aim and Objectives

The aim of the study is to develop a Geosttistical model (Regression Kriging Model) that could be used for prediction of mean surface temperature of Nigeria.

The objectives of the study are to:

1. Produce surface maps of mean surface temperature of Nigeria.
2. Develop a regression kriging model that could be used for prediction of mean surface temperature of Nigeria.
3. Compare the performance of regression kriging to multiple linear regression and ordinary kriging.

1.4 Significance of the Study.

The upward trend in the surface temperature of Nigeria and its consequences as widely reported by researchers is not strange. The result of this work shall provide a frame work for appropriate methodology to both old and intending researchers of temperature in Nigeria. Also, from the works reviewed, much is not yet done using geostatistics as a recommended tool for geographically referenced data in Nigeria and this work shall be of great help to researchers in seeing the usefulness of geostatistics.

1.5 Study Area and Data Used.

Nigeria with an area of 909,890 square kilometres is situated between Longitude 3° and 14° East and Latitude 4° and 14° North. Nigeria is also blessed with favourable and varied climatic conditions. Temperature in Nigeria varies according to the season of the year as with other lands found in the tropics. Nigeria’s seasons are
determined by rainfall with rainy season and dry season being the major seasons in Nigeria (nbs abstract, 2008). The study covered readings of average surface temperature by Meteorological and Hydrological service stations in Nigeria as contained in annual reports of National Bureau of Statistics (NBS), 2009. The data were used to model mean surface temperature as a function of mean relative humidity, mean radiation, elevation, evaporation, distance of state capitals of Nigeria to Lagos (nearby ocean/sea), longitude and latitude (position of each of the monitoring stations). The choice of variables selected above are some out of factors that affect surface temperature.

Figure 1.1: Map of Nigeria showing Meteorological and Hydrological Service Stations

Table 1.1: Sample Averages of Monthly Climatic Elements for the Period 2006-2009.

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1.6 Definition of terms.

**Temperature**: Is a measure of the degree of hotness or coldness of a body.

**Geostatistics**: A branch of statistics that specializes in the analysis and interpretation of any spatially (and temporally) referenced data, with a focus on inherently continuous features (Isah, 2013). A quantitative measure of spatial correlation used for estimation and simulation (Deutsch and Journel, 1992)

**Relative humidity**: The amount of moisture in the air.

**Elevation**: The height of a given location above the sea level.

**Slope**: is the sum of the regression coefficients of the fitted MLR model.

**Lag**: is the separation distance between pairs of locations and is symbolized as $h$.

**Surface Temperature**: Measurement of temperature taken on the earth surface or at a height slightly above the earth surface.

**Latitude**: A geographic coordinate that specifies the North-South position of a point on the earth’s surface.

**Longitude**: The angular distance East or West of the prime meridian that stretches from the North Pole to South Pole and passes through the Greenwich Meridian.

**Experimental Variogram**: An experimental variogram is a plot showing how one half the squared differences between the sampled values (semivariance) changes with the distance between the point-pairs.

**Prediction**: Is inference about the unobserved values of a target variable.
**Biotic Factors:** These are various types of living organisms, from microbiological to higher animal and plant species that influence surface temperature of a region.

**Stationarity:** This is a phenomena used to describe similarity in statistical property of a target variable (constant mean and variance).

**Autocorrelation:** Correlation between elements of a series and others from the same series separated from them by a given interval.

**P-Value:** A p-value measures the strength of evidence in a support of a null hypothesis.

**Scatter Plot:** Is a graphic tool used to display the relationship between two quantitative variables.

**Parameter:** A quantity used to describe a population; it can be described as a population quantity which value is estimated from the available sample information.

**Model:** Graphical, mathematical or verbal representation of a concept, relationship, structure or an aspect of real world.

**Isotropy:** The property of having the same value when measured in different directions.

**Anisotropy:** Is a property of being directionally dependent, which implies different values at different directions.

**Outliers:** A statistical observation that is markedly different in value from the others of the same sample.

**Residual:** The difference between the observed and the predicted measured values.

**Regression:** Is a cause and effect relationship between dependent and independent variables: the independent is the cause while the dependent is the effect.
**Correlation:** Is a measure of the strength of association between two or more variables.

**Approximately, Normally, Distribution:** Approximately, Normally, Distribution of data is one in which the majority of the data points are relatively similar, occurring within a small range of values while there are fewer outliers at the higher and lower ends of the range of data.

**Coefficient of determination;** $R^2$: Is a ratio of explained variation to unexplained variation.

**Euclidean Distance:** The shortest distance between two given locations.
CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The weather has long been a subject of widespread data collection, analysis and interpretation (IPCC, 2007). Precise measurement of air temperature data became possible in the mid-1700s with Gabriel Fahrenheit’s invention of the first standardized mercury thermometer in 1714 (NASA, 2015). Climate change has become more threatening to the sustainable development globally. The mean global temperature have been increasing in line with precipitation increases since 1850, mainly due to the accumulation of greenhouse gases in the atmosphere (Dodson and Mark, 1997).

2.2 Review of related works (Conceptual)

2.2.1 Geostatistics

Kriging has for many decades been used as a synonym for geostatistical interpolation. It originated in the petroleum and mining industry in the early 1950’s by Danie Krige and the statistician H. S. Sichel. The technique was first published in Krige (1951), but it took almost a decade until a French mathematician G. Matheron derived the formulae and basically established the whole field of linear geostatistics (Cressie, 1990; Webster and Oliver, 2001). Different interpolation methods have been used to model the spatial distribution of surface temperature; the most widely used being the inverse distance interpolation weighting, Voronoi tessellation, regression analysis and more recently, geostatistical methods (Hengl, 2009). Sánchez-Palomares et al., (1999) developed maps of climate variables with
2.2.2 Causes of Change in Climatic Variables.

Isah and Usman (2015) adduce to anthropogenic activities as the major factors responsible for variation in micro climate, vegetation and ecosystem.” That man’s
interaction with his environment has been recognized as the major determinant in the modification of the biosphere”. Human effect such as rapid growth in population and economic output per capita in this industrial era are the sources of most contemporary changes in the state and flow of the biosphere and thereby have great consequences on the global environment (Olusina and Odumade, 2012).

Urbanization is most responsible in transformation of the natural landscape through construction of mass buildings and impervious surfaces in varieties of shapes and orientation and thereby alters the local climate (Olea, 2005). The urban environment with its anthropogenic activities contribute to the reduction in outgoing long wave radiation by hindering the loss of sensible heat (Olusina and Odumade, 2012) which result in the build-up of ambient land surface temperature in the urban Centre of 2-3 degrees higher than the sub-urban environment where there is relatively greater cover of vegetation, cultivated lands as well as greater areas of wet soil (Fernando et al., 2007). Scientists believed the earth is currently facing a period of rapid warming brought on by rising levels of heat trapping gasses known as greenhouse gasses in the atmosphere (IPCC, 1997). Greenhouse gasses occur naturally, and without them the planet would be too cool to sustain life. Since the beginning of industrial revolution in the mid-1700s, however, human activities have added more of these gasses into the atmosphere (Ishida, 1993).

National Aeronautics And Space Administration (NASA) Goddard Institute For Space Studies Scientists (2015) reported that the average temperature for 2013 tie with 2009 and 2006, being the seventh warmest years since 1880. That with exception of 1998, the 10 warmest years in the 134 years record fall within the 21st century with 2010 and 2005 ranked as the warmest years on record. NASA (2015) further emphasized that the weather pattern will cause fluctuations in average temperature from year to year. That each successive year will not necessarily be warmer than the year before it but with the current level of greenhouse gasses emission, they expect each successful year to be warmer.
The fourth assessment report of IPCC (2007) indicated an upward trend in global average temperature since 1850 and recognized that 11 of the 12 years (1995-2006) before the publication of the report were ranked among the warmest years in instrumental record (Blennow K., 1998). Unless, global actions aimed at reducing the causes of global warming are taken, a predicted rise in temperature beyond this century is envisaged (Dodson and Mark, 1997). In most parts of the world, both maximum and minimum temperatures have increased with significant positive trends beyond that predicted to be caused by natural variability (Ayalew, L et al.; 2002). The regional projections of the IPCC revealed that the West African region is likely to sustain a 3-4°C increase in temperature over the period 1900-2100. Within this region a 3°C rise is predicted to occur in the coastal and equatorial areas, while the western Saharan region is predicted to have an increase of 4°C (Fernando et al., 2007).

2.2.3 Effects of Climate Change on Environment.

Accurate understanding of climate variation is of paramount importance to recognizing and understanding their effects on humans and environment. NASA (2015) further emphasized that the weather pattern will cause fluctuations in average temperature from year to year. That each successive year will not necessarily be warmer than the year before it but with the current level of greenhouse gasses emission, they expect each successful year to be warmer. The climate variability impact at regional and sub-regional and ecosystem levels is likely to be uneven and unpredictable (Barry and Chorley, 1987). Spatio-temporal climate trend analysis detects and describes pattern, variability, anomaly and trend in space and time. This will further enhance our insight in to the possible natural disasters: bad weather, poor agricultural yields, plants and animals extinction, etc, (IPCC, 2007). Global historical climatology network (NASA, 2015) reported that the combined average temperature over land and ocean surfaces for August, 2015 was 0.88°C (1.58°F) above the 20th Century aver-
age of 15.6°C (60.1°F) and the highest August in the 136 years record. This value surpassed the previous record set in 2014 by 0.09°C (0.16°F). NASA report further revealed that most of the world surfaces substantially get warmer than the average. In some locations, record of August 2015 contributed to the monthly global record warmth “that this was the six month in 2015 that has broken its monthly temperature record (February, March, May, June and July)”. August 2015 also tie with January 2007 as the third warmest monthly departure from average for any of the 1628 months since records began in January 1880, behind February and March 2015 (0.89°C). Five out of the ten monthly temperature departures from average occurred in 2015.

2.2.4 Sources of Spatial Variability

Spatial variability of environmental variables is commonly a result of complex processes working at the same time and over a long period of time, rather than an effect of a single realization of a single factor. Many environmental variables vary not only horizontally but also with depth, not only continuously but abruptly and the variable(s) can be viewed as a signal process consisting of three components (Hengl, 2009): the deterministic component, the spatially correlated component and the pure noise-partially micro scale variation, partially the measurement error.

2.2.5 Aspects of Spatial Variability

According to Hengl (2009), there are three aspects of Spatial Variability: Geographical Variability, vertical Variability and temporal variability.

1. Geographical variation (2D): The results of spatial prediction are either visualized as 2D maps or cross sections. Some environmental variables such as thickness of soil horizon or soil types do not have a third dimension, that is, they refer to the
earth surface only.

2. Vertical variation (3D). Many environmental variables vary with depth and altitude above the ground surface. Consider variables such as temperature, transition between different soil layers can also be both gradual and abrupt, which require a double-mixed model of soil variation for 3D spatial prediction.

3. Temporal variation. Environmental variables connected with distribution of plant and animal species vary not only within a season but also within few moments. Even soil variables such as pH, nutrients and water saturation levels and water content, can vary over a few years, within a single season or even over a few days (Hengl, 2009).

2.2.6 Spatial prediction Models.

The variability of environmental variable(s) can be determined by a finite set of inputs and they exactly follow some known physical rule. If the formula is known, the value of the target variable can be predicted exactly.

Spatial prediction models can be classified according to the amount of expert knowledge included in the analysis: mechanical, linear statistical and expert based models.

a. Mechanical (Deterministic) Models: these are models where arbitrary parameters are used and no estimate of the model error is available and usually, no strict assumptions about the variability of the feature exist. Such models include: Thiessen polygon, inverse distance interpolation, regression on coordinates, Splines and Natural neighbour.

b. Linear statistical (probability) models: the model parameters here are estimated in objective way, following probabilistic approach. The predictions are accompanied with an estimate of the prediction error. Such models include kriging, environmental correlation (regression based), Bayesian based models (Bayesian maximum entropy)
and hybrid models (regression kriging).

c. Expert Based System: These are knowledge-driven expert system (for example, hand-drawn maps), data-driven expert system (for example neural networks) and machine learning algorithms (purely data-driven);

2.2.7 Geostatistical Mapping.

Measurements taken on Earth and environmental sciences have a spatiotemporal reference. A spatiotemporal reference is characterized by the following parameters (Hengl, 2009):

i. Geographic location (longitude and latitude or projected X, Y coordinates)

ii. Height above the ground (elevation)

iii. Time of measurement (yearly, hourly, daily, etc.)

iv. Spatiotemporal support (size of the block of material associated with measurement, time interval of measurement.)

As recommended by (Olusina and Odumade, 2012), Amadi et al., 2014 and other researchers on climate change, doing our research in 2016 and 2017 is as good as filling the gap.

2.3 Empirical Literature

Atkinson and Lloyd (1998) carried out spatial interpolation of climate variables in Switzerland, using two variables: temperature and rainfall. In their work they compared different kriging methods and recommended Indicator kriging and the Ordinary Kriging methods as the right interpolation methods to be used.
Olea R.A.,(1999) carried out spatial interpolation of climate variables in the South Eastern Nigeria using five variables and suggested Ordinary kriging with global climatological variogram and Ordinary Kriging with phase variogram as the appropriate interpolation methods for the zone.

Todd and Dean (2002), in their work titled spatial estimation of air temperature differences for landscape-scale studies in mountain environment in Oregon, Western Cascades. They considered a nested series of temperature regression models. They began with a simple elevation model, in which they collected data to develop a site specific lapse rate—a quantitative description of decrease in temperature with increase in elevation. Model 1 did a reasonable job of fitting the data but with 19 of the 45 points over or under estimated by 0.5°C. Adding distance to stream reduced the number to 10. Adding radiation to the model reduced the number to 6, until additional variable do not longer significantly improve the model. Model 2 was better than model 1, F-statistics was 39.79, P < 0.001. Model 3 was able to describe the spatial variability in temperature slightly better than model 2 with F-statistics was 3.8, P-value was 0.008.

Fernando et al., (2007) carried out geostatistical modelling of air temperature in mountain region of Northern Spain. They compared five geostatistical and two regression models using data obtained from the coolest and the warmest months (January and August). The independent variables used were: altitude, latitude, distance to nearby sea and solar radiance. They adduce that ordinary kriging with external drift was more satisfactory than the ordinary kriging and that a complex regression model which included latitude, distance to the sea and solar radiance as independent variables showed better results in terms of mean absolute error.

Hengl (2009) modelled the surface temperature of Croatia using Digital Elevation Model (DEM), latitude, longitude and distance to sea as explanatory variables. All the variables had over 50% R² as one of the performance indicators and were in cooperated in to the model.
Olusina and Odumade., (2012) modelled climatic variation parameters of Nigeria using Statistical down scaling approach and reported that the upward trend in the mean maximum air temperature and precipitation of Nigeria will continue till 2050. They equally remarked that the north will experience higher temperature increase than the south while the southern part of the country will experience higher precipitation than the northern part.

Isah (2013), in his work titled Multivariate spatial modelling and prediction of meteorological data, compared cokriging and the multiple linear regression models and suggested cokriging for multivariate spatial modelling and prediction of meteorological data.

Amadi et al., (2014). In their work titled trends and variations of monthly mean minimum and maximum temperature over Nigeria for the period 1950-2012. They aimed at revealing spatial and temporal pattern of long term trend in temperature of Nigeria. A time series plot, correlation, descriptive statistics and Mann-kendall tests were used for the analysis. The minimum temperature had higher trends coefficients than the maximum temperature and the interstation spatial coherence revealed by correlation coefficients indicated that both the minimum and the maximum temperatures were positively correlated while the Mann-kendall tests showed a general warming trend across the nation Nigeria.

Isah and Usman (2015) carried out analysis of water quality parameters in rivers and streams in Niger state. They considered parameters such as: Total hardness (TH), Total dissolved solids(TDS), Total Colifor (TCO), E-Coli and Magnesium. The Ordinary kriging was used to determine the spatial continuity of river water parameters and different semivariogram models were tested and the Linear yielded the best amongst others on cross validation. Their results further showed that water quality in dry season changes rapidly than it does in the raining season.

Many regional studies have shown positive trend in temperature, though the changes
varies slightly from region to region (GISTEMP, 2015; Dudson and Marks, 1997; Goodale et al.; IPCC, 2007; Lapen and Hayhoe, 2003).
CHAPTER THREE

METHODOLOGY

3.1 Introduction.

In this study, four years of measurements of monthly mean surface temperature in Nigeria provided by National Bureau of Statistics (NBS) from the Nigerian Meteorological and Hydrological Service for the period 2006-2009 were used to model temperature as a function of mean relative humidity (MRH), mean radiation (MRAD), elevation, evaporation, distance of state capitals of Nigeria to Lagos (nearby ocean/sea), longitude and latitude (position of each of the state capitals. To achieve the aim of the study, three approaches: Multiple Linear Regression, Ordinary Kriging and Regression Kriging were adopted and provision for comparing the performances of the three approaches were provided.

3.2 Linear Statistical Prediction Models

These are models which predictions are based on stringent assumptions. These models are Kriging (plain geostatistics), regression based models, Bayesian based models and hybrid models. Kriging is one of the spatial prediction models. It originated from the mining Engineer Danie Krige and the Statistician Sichel (1950). The technique was first published in Krige (1951) and the formulae were derived by Matheron (1960) which basically formed the field of linear geostatistics (Hengl, 2009).

3.2.1 Regression model.

The regression model employs a family of function known as generalized linear models which assumes a linear relationship between the inputs and the outputs.
The regression coefficients are the output from the model fitting process; Draper and Smith (1998).

A common regression based approach is the multiple linear regression model stated by Kutner and Neter (2004) as:

\[ T = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \mu = \sum_{k=0}^{p} \beta_k X_k + \mu; \quad X_0 = 1 \tag{3.1} \]

Where; \( X_k \) denote independent variables at target location; \( P \) denote the number of independent variables, \( \mu \) the random error term that capture the effect of omitted values while \( \beta_k \) denote regression coefficients.

### 3.2.2 Estimation of Parameter.

The coefficients (\( \beta_k \)) of the independent variables were obtained by using the Ordinary least squares as given in equation (3.2) below.

\[ \beta^\wedge = (X^t X)^{-1} X^t T \tag{3.2} \]

Where; \( X \) denote matrix of predictors of order \( np + 1 \), \( T \) denotes vector of sampled observations.

### 3.2.3 Assumptions of Multiple Linear Regression Model

1. The relationship between temperature values and the independent variables values is linear and additive.

- The expected value of temperature is a straight line function of each independent variable holding the others fixed.
- the effects of the different independent variables on the expected value of the temperature are additive.

To detect non linearity and non-additive, a plot of observed versus predicted or residual versus predicted, the points should be symmetrically distributed around a diagonal line or around the horizontal line in the later. The log-transformation is used to fix non-linearity and additive.

2. Statistical independence of the error (no correlation between consecutive errors in the case of time series data). To detect statistical independence, residual time series plot (residual versus row data or plot of residual autocorrelation). The Durbin Watson statistic provides a test for significant autocorrelation.

3. Homoscedasticity: (constant variance) of the error versus any independent variable

Violation of homoscedasticity (heteroscedasticity) makes it difficult to gauge the true standard deviation of the forecast errors, resulting in the confidence intervals that are too wide or too narrow. A Plot of residuals versus time, for a time series data is used to detect this effect and to fix that, a log transformation of the dependent variable is appropriate.

4. The errors are approximately, normally distributed. If the error distribution is significantly non normal, the confidence interval may be too wide or too narrow. A normal probability plot of the residuals is used to test for normally distributed error. Other tests include; Kolmogorov Smirnov test, Shapiro wilk test, Jarque Bera test, and the Anderson-Darling test. To fix non normality, a log transformation is appropriate
3.2.4 The prediction error

The prediction error of a multiple linear regression model is given by Hengl (2009) as:

\[ \hat{\delta}^2_{OLS}(S_0) = \frac{MSE}{1 + X_0'(X'X)^{-1}X_0} \]  

(3.3)

Where; MSE denote mean square error around the regression line, X is the matrix of predictors and \( X_0 \) denote vector of independent variables at unvisited locations.

Euclidean distance is stated as :

\[ d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]  

(3.4)

3.3 Geostatistical Methods

Geostatistics is a valuable tool that can be used to characterize spatial, temporal or spatiotemporal phenomena. That is, each data value is associated with a location in space and there is at least an implied connection between the location and the data value. Location here could have two different meanings: One is simply a point in space which only exists in an abstract mathematical sense while the second is an area or volume in space (average value of an observed value).

3.3.1 Experimental Variogram

The experimental variogram measures the average degree of dissimilarity between un-sampled values and a nearby data value and consequently can depict autocorre-
lation at various distances (Robinson and Metternicht, 2006):

\[ 2\hat{\gamma}(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} [T(x_i) - T(x_i + h)]^2 \]  

(3.5)

where \( N(h) \) is the number of data pairs with a given class of distance and direction.

If the values at \( T(x_i) \) and \( T(x_i + h) \) are auto-correlated, the result of equation (3.5) relative to an uncorrelated pair of point will be small. Using an analysis of the experimental variogram, a suitable model (Gaussian, linear, exponential or spherical) is fitted. This is made using weighted least square and relevant parameters of the covariance structure (range, nugget and sill) are then used in the kriging procedure. Once the variogram model is estimated, it is used to derive semivariances at all locations and solve the kriging weights. When observations are available at all directions and specified distances, the anisotropy may be modelled directly from the directional experimental variogram model but when elevation is included in the kriging system, the directional variograms can show zonal anisotropy (Hengl, 2009).

### 3.4 Semivariogram

The novelty that Matheron (1962) and colleagues introduced to the analysis of point data is the derivation and plotting of semivariances (differences between the neighbouring values);

\[ \hat{\gamma}(h) = \frac{1}{2} \sum \left[ (T(S_i) - T(S_i + h))^2 \right] \]  

(3.6)

Where; \( T(S_i) \) denote temperature at sampled locations and \( T(S_i + h) \) denote value of neighbour at distance \( (S_i + h) \). Alternatively;

\[ \text{Semivariance} = \text{Slope} \,(\text{distance}) \]  

(3.7)
3.4.1 Binning

With larger dataset the number of pairs of locations will increase rapidly and will quickly become unmanageable. Therefore, pairs of locations are grouped, which is referred to as binning. A bin is a specified range of distances. That is, all points that are within range are grouped into the first bin, those that are within the next range are grouped into the second bin, so on and so forth. The average semivariance of all pairs of points is obtained after grouping the observations into bins.

The kriging weights are derived from a statistical model of spatial correlation expressed as semivariograms that characterized the spatial dependency and structure in the data. The OK weights are solved by multiplying the covariances between sampled and unsampled locations:

\[ \lambda_o = c^{-1} \nu_o; c(|h| = 0) = c_0 + c_1 \]  

(3.8)

Where \( c \) is the covariance matrix derived for \( n \times n \) observations and \( \nu_o \) is the vector of covariances at a new location (Hengl, 2009). The solution to the minimization, constrained by unbiasedness, gives the kriging equation (Hengl, 2009):

\[ \Gamma^{-1} \ast \ell = \lambda \]  

(3.9)

\[ \sum_{i=1}^{n} \lambda_i = 1 \]  

(3.10)

\[
\begin{bmatrix}
  c(s_1, s_1) & \cdots & c(s_1, s_n) & 1 \\
  \vdots & \ddots & \vdots & \vdots \\
  c(s_n, s_1) & \cdots & c(s_n, s_n) & 1 \\
  1 & \cdots & 1 & 0 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
  c(s_o, s_1) \\
  \vdots \\
  c(s_o, s_n) \\
  1 \\
\end{bmatrix} =
\begin{bmatrix}
  w_1(s_o) \\
  \vdots \\
  w_n(s_o) \\
  \varphi \\
\end{bmatrix}
\]

(3.11)

where \( \varphi \) is the so-called Lagrange multiplier. Also, \( C \) is a matrix of \( (n + 1) (n + 1) \) dimension, since it is used to derive kriging weights. One extra row and column are added to ensure that the sum of weights is equal to unity.
3.4.2 The Ordinary Kriging Model.

The Ordinary Kriging model according to Matheron (1962) is given by:

\[ T(s_0) = \mu + e(s) \]  \hspace{1cm} (3.12)

Where; \( \mu \) denote stationary function, \( e(s) \) denote spatially correlated stochastic part of variation, \( T(s_0) \) denote temperature at location ‘\( s_0 \)’.

3.4.3 Assumptions of Ordinary kriging.

(i) Trend function is constant (stationary in mean, that is, \( \mu \) is constant)

(ii) The Variogram is constant in all area of study.

(iii) The temperature values are approximately, normally distributed.

3.4.4 The Ordinary Kriging prediction

Matheron (1962) stated that the ordinary kriging prediction is given by:

\[ \hat{T}_{OK}(S_0) = \sum_{i=1}^{n} W_i(S_0) T(S_i) = \lambda_0 T \]  \hspace{1cm} (3.13)

Where; \( \lambda_0 \) denote vector of kriging weights \( (W_i) \) and \( T(S_i) \) denote vector of \( n \) observations (temperature values) at primary locations.

3.4.5 The prediction variance

According to (Matheron,1962), the prediction variance of Ordinary kriging (estimated variance of the prediction error) is defined as the weighted average of the covariances from the new point \( (S_0) \) to all calibration points \( (X_1, \ldots, X_n) \), + the
Lagrange multiplier.

\[ \hat{\delta}^2_{OK} (S_0) = C_0 + C_1 + \sum_{i=1}^{n} W_i (S_0) C \left( S_0, S_i \right) + \varphi \]  \hspace{1cm} (3.14)

Where; \( C \left( S_0, S_i \right) \) denote the covariance between the new location and the sampled point pairs, \(( C_0 + C_1)\) the sill, \( W_i \left( S_0 \right) \) the kriging weight at location \( S_0 \), \( C_1 \) is the sill parameter and \( \varphi \) denote the Lagrange multiplier.

### 3.4.6 Ordinary kriging with Zonal Anisotropy

When observations are available at all directions and specified distances, the anisotropy may be modelled directly from the directional experimental variogram model but when elevation is included in the kriging system the directional variograms can show zonal anisotropy. Both directional variograms can be modelled as the sum of the best fitted which constitutes a valid model (Hengl, 2009).

### 3.5 Regression Kriging Model

Many geostatisticians believed that there is only one best linear unbiased prediction (BLUP) model for spatial data from which all other linear techniques can be derived (Hengl, 2009). The generic mapping technique is Regression Kriging. The Ordinary Kriging and other forms of kriging and the Multiple Linear Regression are special cases of the Regression Kriging. Regression Kriging is a hybrid model (a combination of regression model and kriging model). The multiple linear regression (MLR) shall be used to treat the deterministic aspect while the Ordinary Kriging (OK) shall be used to treat the spatially stochastic correlated component of the Regression Kriging in this study.

Matheron (1969) stated that the temperature at some locations can be modelled as
a sum of deterministic and stochastic components, given as:

\[ T(s) = m(s) + k(s) + e(s) \]  \hspace{1cm} (3.15)

Where; \( m(s) \) denote deterministic part; \( k(s) \) denotes spatially correlated stochastic component and \( e(s) \), the error term.

### 3.5.1 Assumptions of Regression kriging.

(i) Temperature values are approximately, normally distributed.

(ii) The residual variogram should be constant at all location.

(iii) Temperature values must be auto-correlated.

(iv) The temperature values must be correlated with the independent variables.

### 3.5.2 The regression kriging prediction.

Matheron (1962) stated regression kriging model as:

\[ T_{RK}(S_o) = \sum_{k=0}^{n} \beta_k X_k(S_o) + \sum_{i=1}^{n} \lambda_i e(S_i); \quad X_o = 1 \]  \hspace{1cm} (3.16)

Where; \( e(S_i) \) is the residual at location \( S_i \), \( \beta_k \) is the deterministic model coefficient, \( X_k \) denote independent variable, \( \beta_0 \) is estimated intercept and \( \lambda_i \) denotes Kriging weights determined by spatial dependence structure of the residual at location \( S_i \).

### 3.5.3 Parameters Estimation

#### 3.4.3.1 Coefficients of the independent Variables

The regression coefficients \( \beta_k \) were estimated from the sample by fitting method using generalised least squares
\( \beta_{GLS}^{\wedge} = (X'C^{-1}X)^{-1}X'C^{-1}T \) (3.17)

Where; \( \beta_{GLS}^{\wedge} \) denote vector of estimated regression coefficients, \( C \) is covariance matrix of residuals, \( X \) denotes matrix of independent variables at sampled locations and \( T \) is vector of measured temperature values.

### 3.4.3.2 Regression Kriging Weight

The Regression Kriging Weights are obtained as given in equations (3.9,3.10 and 3.11)

### 3.5.4 Regression Kriging Prediction Variance

Matheron (1962) stated that the BLUP has a prediction variance as given in equation (3.20):

\[
\delta_{RK}^2 (S_0) = (C_0 + C_1) - C_0 C^{-1} C_0 + \left( X_0 - X'C^{-1} C_0 \right)^\prime \left( X'C^{-1} X \right)^{-1} \left( X_0 - X'C^{-1} C_0 \right)
\]

(3.18)

Where; \( (C_0 + C_1) \) denote sill, \( C_1 \) is Sill parameter and \( C_0 \) denote vector of covariance of residuals at unvisited Site, \( C \) is the covariance matrix of residuals and \( X \) is a matrix of auxiliary variables at the sampled locations.

### 3.5.5 Hengl’s Regression Kriging model

Hengl (2009) proposed a Regression Kriging Model as:

\[
LST (S_0) = b_0 + b_1.DEM (S_0) + b_2.LAT (S_0) + b_3.DIST (S_0) + b_4.LONG (S_0) + \sum_{i=1}^{4} \lambda_i.e (S_i)
\]

(3.19)
Where; $LST(S_0)$, DEM., LAT., DIST., and LONG., denote land surface temperature at given location and time, digital elevation model, latitude, distance to nearby stream and longitude at location $S_0$.

### 3.5.6 Prediction Variance

Matheron (1962) stated that the BLUP has a prediction variance as given in equation (3.20):

$$
\delta^2_{RK}(S_0) = (C_0 + C_1) - C_0^{-1}C_0 + \left( X_0 - X' C_0^{-1} C_0 \right)' \left( X' C_0^{-1} X \right)^{-1} \left( X_0 - X' C_0^{-1} C_0 \right)
$$

(3.20)

Where; $(C_0 + C_1)$ denote sill, $C_1$ is Sill parameter and $C_0$ denote vector of covariance of residuals at unvisited Site, $C$ is the covariance matrix of residuals and $X$ is a matrix of independent variables at the sampled locations.

### 3.6 The proposed geostatistical model of mean surface temperature of Nigeria.

$$
T(S_o) = \sum_{k=0}^{7} \beta_k X_k(S_o) + \sum_{i=1}^{6} \lambda_i e(S_i)
$$

(3.21)

Where; $\lambda_i$, $\beta_K$, $X_k(S_0)$, $e(S_i)$ and $T(S_o)$ denote the kriging weights, the regression coefficients, matrix of independent variables at sampled locations, residual at location $S_i$ and the mean surface temperature respectively.

### 3.6.1 Assumptions of the proposed model

The assumptions of the Proposed Regression Kriging Model are similar to Matheron (1962) Regression Kriging Model assumptions. They are:
(i) Temperature values are approximately, normally distributed.

(ii) Temperature values do not need to be stationary but its residual should be stationary.

(ii) The residual variograms should be constant at all locations.

(iii) Temperature values must be auto-correlated.

(iv) The temperature values must be correlated with the auxiliary variables.

3.7 Relationship of Regression Kriging to Multiple Linear Regression

According to Hengl (2009), if the residuals show no spatial autocorrelation (pure nugget effect), the Regression Kriging prediction converges to pure MLR because the covariance matrix \( C \) becomes identity matrix (Hengl, 2009):

\[
\begin{pmatrix}
C_0 + C_1 & \cdots & 0 \\
\vdots & C_0 + C_1 & 0 \\
0 & 0 & C_0 + C_1
\end{pmatrix} = (C_0 + C_1) \cdot I \tag{3.22}
\]

So the kriging weights at any location predict the mean residual zero.

Similarly, the regression kriging Variance reduces to the MLR Variance

\[
\hat{\delta}_{RK}^2 (S_0) = (C_0 + C_1) - 0 + X_0^t \left( \frac{1}{(C_0 + C_1)^2} \right)^{-1} X_0
\]

\[
\hat{\delta}_{RK}^2 (S_0) = (C_0 + C_1) + (C_0 + C_1) X_0^t (X^t X)^{-1} X_0
\]

And since \((C_0 + C_1) = C(0) = MSE\), the RK variance reduces to MLR variance. That is,

\[
\begin{aligned}
\frac{\hat{\delta}_{RK}^2}{\hat{\delta}_{OLS}^2} (S_0) &= MSE \left[ 1 + X_0^t (X^t X)^{-1} X_0 \right] \tag{3.23}
\end{aligned}
\]
3.8 Relationship of Regression Kriging to Ordinary Kriging.

According to Hengl (2009), if the temperature values show no correlation with the independent variables, the Regression Kriging model reduces to the Ordinary Kriging model because the deterministic part is equal to the global mean.

\[
\hat{T}(S_0) = \mu = \frac{1}{n_k} \sum_{i=1}^{n_k} T(S_i)
\]  

(3.24)

Also from equations (3.15) and (3.12)

\[
T_{RK}(s) = \mu + k(s) = T_{OK}(s)
\]  

(3.25)

From the above, it implies that:

\[
M(S) + e = \mu
\]  

(3.26)

3.9 Cross Validation

The common validation method in climatological studies has been variously termed as cross validation (Nalder and Wein, 1998). Cross validation (leaving-one-out method) is based on removing one data point at a time and performing the interpolation for the location of the removed point using the remaining samples. At the final step of cross validation, the difference (residual) between observed and predicted values of the point are calculated. The overall performance of each kriging method is calculated using mean prediction error (MPE), mean standard prediction error (MSPE), average kriging standard error (AKSE) and Root mean square prediction error (RMSPE):

\[
MPE = \frac{1}{N} \sum_{k=1}^{n} (T_{OK} - T_{PK})
\]  

(3.27)
\[ MSPE = \frac{1}{N} \sum_{k=1}^{n} \frac{(T_{0K} - T_{PK})}{\delta(K)} \]  

(3.28)

\[ AKSE = \left[ \frac{1}{N} \sum_{k=1}^{n} \delta(K) \right]^{\frac{1}{2}} \]  

(3.29)

\[ RMSPE = \left[ \frac{1}{N} \sum_{k=1}^{n} (T_{OK} - T_{PK})^2 \right]^{\frac{1}{2}} \]  

(3.30)

Where; \( T_p, T_0 \) and \( T_m \), are the predicted, observed and the mean values of temperature while \( p \) is the total number of auxiliary variables and \( n \) is the sample size.

\( T_{OK} \) denote Observed temperature at location \( K \), \( T_{PK} \) is the Predicted temperature at location \( K \), \( N \) is the Number of pairs of observed and predicted values and \( \delta(K) \) the Prediction standard error at location \( K \).

As an indicator of prediction error, the MPE and MSPE values reveal the degree of bias in model prediction and should be close to zero. Assessment of variability in prediction, the RMSPE and AKSE values show the precision of predictions and should be equal to one another. In other words, the RMSPE reveals the level of scatter that a model produces and provides a comparison of the absolute deviation between the predicted and the observed values. The lower the RMSPE values, the better a model is indicated to perform.

Over estimation of variability occurs when \( AKSE > RMSPE \) and under estimation occurs when \( AKSE < RMSPE \).

3.10 Software to be used for the Analysis

The R software will be used for cross-validation and the GS+ will used for variogram analysis.
CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction.

Different individuals take different approaches to the analysis and interpretation of data and this may yield different results. The differences in the results could be attributed to differences in the choice of methodology, data interpretation and in few cases, error in procedure (Hengyl, 2009). This chapter therefore focuses on the analysis and interpretation of results therein. The analysis followed the same sequence of statistics given in the preceding chapter.

4.2 Preliminary Statistical Analysis

To develop mean surface temperature (MST) model using geostatistics procedures, the MST values were tested for normality by inspection of autocorrelation and histogram plots as in Figure 4.11. Looking at the autocorrelation (acf) plotted on MST values, it showed that the temperature values are spatially auto correlated and therefore possible to predict the value at one location based on the value sampled from a nearby location. The Kolmogorov-Smirnov and Shapiro-Wilk tests for Normality revealed that both tests were significant at 5% level of significance. That is, the MST data are approximately, normally distributed.

Table 4.1: Tests for Normality

<table>
<thead>
<tr>
<th>Kolmogorov-Smirnov</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistic df Sig.</td>
<td>Statistic Df Sig.</td>
</tr>
<tr>
<td>MST .06 148 .02</td>
<td>.98 148 .01</td>
</tr>
</tbody>
</table>

34
4.3 Regression models

The relationship between MST and independent variables in the OLS regression models were fitted using equation 13 and the result presented in Table 1. The result shown that Mean Relative Humidity and Distance contributed negatively while, Elevation, Mean Radiation, Evaporation, Longitude and Latitude contributed positively to the MLR model, though the coefficients are significant and explained 87% of variability in the MLR model. A further question of interest is whether any of the observations greatly affect the estimates. The MLR fitted to the full dataset produced a set of four plots: residuals versus fitted values, a Q-Q plot of standardized residuals, a scale-location plot (square roots of standardized residuals versus fitted values, and a plot of residuals versus leverage that adds bands corresponding to Cook’s distances figure 2. Cook’s distance is a measure of how much the estimate changes as each observation is dropped. It was found that 62, 74 and 99 were outliers and after dropping the three outliers the MLR fitted is given in Table 4.2:

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t- value</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-42.804</td>
<td>6.878</td>
<td>-6.223</td>
<td>0.000</td>
</tr>
<tr>
<td>Longitude</td>
<td>0.107</td>
<td>0.046</td>
<td>2.344</td>
<td>0.046</td>
</tr>
<tr>
<td>Latitude</td>
<td>0.002</td>
<td>0.046</td>
<td>0.045</td>
<td>0.964</td>
</tr>
<tr>
<td>MRH</td>
<td>-0.124</td>
<td>0.155</td>
<td>-0.800</td>
<td>0.425</td>
</tr>
<tr>
<td>MRAD</td>
<td>1.188</td>
<td>0.144</td>
<td>8.266</td>
<td>0.000</td>
</tr>
<tr>
<td>ELEV</td>
<td>0.300</td>
<td>0.269</td>
<td>1.114</td>
<td>0.268</td>
</tr>
<tr>
<td>EVAP</td>
<td>0.310</td>
<td>0.053</td>
<td>5.847</td>
<td>0.000</td>
</tr>
<tr>
<td>DIST</td>
<td>-0.333</td>
<td>0.402</td>
<td>-0.830</td>
<td>0.408</td>
</tr>
</tbody>
</table>

Some of the variables are significant at $p = 0.05$. R-squared: 0.876, Adjusted R-squared: 0.8693, Residual standard error = 4.664, F-statistic: 4.864 .
4.4 Results of Geostatistical Models

In order to fit the kriging model, it is sufficient to fit the variogram for the residuals. The parameters for the fitted theoretical variogram models are shown in Table 4.3. The Gaussian model explained the highest variability of \( R^2 = 63\% \) of the fitted isotropic variogram model for MST; while exponential model explained the highest variability of 51% in the MST values of the fitted anisotropic variogram models. The Gaussian and exponential models were therefore chosen as the most reliable models for interpolating MST and used for the final map productions. The two models had the smallest residual sum of squares (RSS), which indicated a tight fit of the models to the MST distributed around 2.03 and 57.4 values respectively.

The fitted experimental variogram of isotropy using Gaussian model, and the four directional variograms in the XYZ plane of anisotropic using exponential model is
Table 4.3: Parameters for the fitted theoretical variogram models for MST

<table>
<thead>
<tr>
<th>Isotropic Variogram</th>
<th>Model</th>
<th>Nugget</th>
<th>Sill</th>
<th>Range (A)</th>
<th>RSS</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.827</td>
<td>1.906</td>
<td>6.355</td>
<td>3.950</td>
<td>0.277</td>
<td></td>
</tr>
<tr>
<td>Spherical</td>
<td>0.001</td>
<td>1.717</td>
<td>3.470</td>
<td>2.050</td>
<td>0.628</td>
<td></td>
</tr>
<tr>
<td>Exponent</td>
<td>0.001</td>
<td>1.749</td>
<td>4.020</td>
<td>2.260</td>
<td>0.608</td>
<td></td>
</tr>
<tr>
<td>Gaussian</td>
<td>0.062</td>
<td>1.676</td>
<td>2.355</td>
<td>2.030</td>
<td>0.629</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Anisotropic Variogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Linear</td>
</tr>
<tr>
<td>Spherical</td>
</tr>
<tr>
<td>Exponent</td>
</tr>
<tr>
<td>Gaussian</td>
</tr>
</tbody>
</table>

Table 4.4: Directional variogram with isotropic and anisotropic models for MST

<table>
<thead>
<tr>
<th>Isotropic Variogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree</td>
</tr>
<tr>
<td>0.001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Anisotropic Variogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree</td>
</tr>
<tr>
<td>0°</td>
</tr>
<tr>
<td>45°</td>
</tr>
<tr>
<td>90°</td>
</tr>
<tr>
<td>135°</td>
</tr>
</tbody>
</table>

In addition, the anisotropic variogram in Figure 4.3 shown that the spatial continuity in the directions of North with angle 0° and North-East with angle 45° were stronger than in the directions East with angle 90° and South East with angle 135°.
In the North and North East directions, the semivariogram levelled off to the sill when it reached a distance of about 9.30 kilometres while in the East and South East directions the spatial correlation for the semivariogram are similar except that it levelled off earlier at about 4.60 kilometres.

![Image](image1)

**Figure 4.2:** Variogram of fitted isotropic model for MST residuals

![Image](image2)

**Figure 4.3:** Variogram of fitted anisotropic models of (i) 0° (ii) 45° (iii) 90° and (iv) 135° for MST residuals
Figure 4.4: (i) OK and (ii) RK of isotropic variogram using Gaussian model: Spatial prediction of MST

Figure 4.5: (i) OK and (ii) RK of anisotropic variogram using exponential model: Spatial prediction of MST
For a comparison, the predictions at all locations of OK and RK were compared side-by-side in figures 4.4 and 4.5. Visually, there are clear differences in the two maps. It could be observed that low MST predictions occurred in the North Central region in OK maps in the model with isotropy and anisotropy as compared to RK models. Since visual comparison may not be enough, the study also used the leave-one-out cross validation method given in table (4.10).

4.5 The Semivariogram

To compute the values for the $\Gamma$ matrix, we examined the structure of the data by creating the semivariogram. In a semivariogram, half the difference squared between the pairs of locations of longitude were plotted relative to the distance {Euclidian distance} that separate them:

<table>
<thead>
<tr>
<th>S/N</th>
<th>Pairs of Sampled Locations (MST Value)</th>
<th>Distance</th>
<th>$\text{Diff}^2$</th>
<th>Semivariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L(1, 2) = 57</td>
<td>6.941</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>L(1, 3) = 58</td>
<td>3.836</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>L(1, 4) = 54</td>
<td>4.178</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>L(1, 5) = 54</td>
<td>4.5</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>L(1, 6) = 55</td>
<td>5.291</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>L(2, 3) = 59</td>
<td>8.034</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>L(2, 4) = 55</td>
<td>8.491</td>
<td>9</td>
<td>4.5</td>
</tr>
<tr>
<td>8</td>
<td>L(2, 5) = 55</td>
<td>11.188</td>
<td>9</td>
<td>4.5</td>
</tr>
<tr>
<td>9</td>
<td>L(2, 6) = 56</td>
<td>8.809</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>L(3, 4) = 56</td>
<td>7.993</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>L(3, 5) = 56</td>
<td>6.974</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>L(3, 6) = 57</td>
<td>9.124</td>
<td>9</td>
<td>4.5</td>
</tr>
<tr>
<td>13</td>
<td>L(4, 5) = 52</td>
<td>4.260</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>L(4, 6) = 53</td>
<td>1.194</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>15</td>
<td>L(5, 6) = 53</td>
<td>5.226</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

With larger dataset the number of pairs of locations will increase rapidly and will quickly become unmanageable. Therefore, pairs of locations are grouped, which is referred to as binning. A bin is a specified range of values. That is, all points that
are within 1 to 3 kilometres apart are grouped into the first bin, those that are within 3 to 5 kilometres apart are grouped into the second bin, and so forth.

The average semivariance of all pairs of points is obtained after grouping the observations into bins and the result is presented in Table 4.6.

### Table 4.6: Binning the Semivariance

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 3</td>
<td>1.194</td>
<td>1.194</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>3 - 5</td>
<td>3.836, 4.178, 4.260, 4.5</td>
<td>4.1935</td>
<td>0.5, 2, 2, 2</td>
<td>1.5</td>
</tr>
<tr>
<td>5 - 7</td>
<td>5.226, 5.291, 6.941, 6.974</td>
<td>6.108</td>
<td>0.5, 0.5, 0.5, 8</td>
<td>2.375</td>
</tr>
<tr>
<td>7 - 9</td>
<td>7.993, 8.034, 8.491, 8.809</td>
<td>8.332</td>
<td>0.5, 2, 4.5, 8</td>
<td>4.25</td>
</tr>
<tr>
<td>9 - 11</td>
<td>9.124</td>
<td>9.124</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>11+</td>
<td>11.188</td>
<td>11.188</td>
<td>4.5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

ΓMatrix (Gamma)

$$
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & & & & \\
2 & 5.562 & 0 & & & \\
3 & 7.578 & 7.672 & 0 & & \\
4 & 8.162 & 11.649 & 12.312 & 0 & \\
1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
$$

Now the Γ matrix and its inverse were used to obtain weights ($\lambda_i$) to assign to the measured values surrounding the prediction location. Thus, $\Gamma^{-1}$ was obtained.
The un-sampled location Abuja was selected amongst other locations for the fact that it is more central.

Next, $\ell$ Vector was created for location 7 (Abuja), given in table 6.

Table 4.7: $\ell$ vector for un-sample location

<table>
<thead>
<tr>
<th>Pairs of Un-sampled and Sampled Locations</th>
<th>Dist.</th>
<th>$\ell$ Vector for Abuja</th>
</tr>
</thead>
<tbody>
<tr>
<td>L(7, 1) = 56</td>
<td>0.92</td>
<td>1.334</td>
</tr>
<tr>
<td>L(7, 2) = 57</td>
<td>6.48</td>
<td>9.396</td>
</tr>
<tr>
<td>L(7, 3) = 58</td>
<td>4.64</td>
<td>6.728</td>
</tr>
<tr>
<td>L(7, 4) = 54</td>
<td>3.52</td>
<td>5.104</td>
</tr>
<tr>
<td>L(7, 5) = 54</td>
<td>4.73</td>
<td>6.859</td>
</tr>
<tr>
<td>L(7, 6) = 55</td>
<td>4.57</td>
<td>6.627</td>
</tr>
</tbody>
</table>

$\Gamma^{-1}$ Matrix

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & -0.099 & & & & \\
2 & 0.0455 & -0.079 & & & \\
3 & 0.023 & 0.021 & -0.068 & & \\
4 & 0.033 & -0.001 & -0.001 & -0.052 & \\
5 & -0.001 & 0.011 & 0.008 & 0.012 & -0.039 & \\
6 & -0.002 & 0.003 & 0.015 & 0.008 & 0.008 & -0.032 & \\
0.047 & 0.153 & 0.062 & 0.197 & 0.230 & 0.310 & -16.057 & \\
\end{bmatrix}
\]
The kriging equation becomes:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & -0.099 & & & & \\
2 & 0.0455 & -0.079 & & & \\
3 & 0.023 & 0.021 & -0.068 & & \\
4 & 0.033 & -0.001 & -0.001 & -0.052 & \\
5 & -0.001 & 0.011 & 0.008 & 0.012 & -0.039 & \\
6 & -0.002 & 0.003 & 0.015 & 0.008 & 0.008 & -0.032 & \\
\end{bmatrix}
\begin{bmatrix}
0.924 \\
-0.508 \\
-0.038 \\
0.060 \\
0.238 \\
0.324 \\
-6.489 \\
\end{bmatrix}
= 
\begin{bmatrix}
1.334 \\
9.396 \\
6.728 \\
5.104 \\
6.859 \\
6.627 \\
-16.057 \\
\end{bmatrix}
\]

4.6 Making a Prediction

Table 4.8 is kriging predictor for the six locations in which measurements were taken.

**Kriging variance** According to Hengl, (2009) the kriging variance of the prediction error is defined as the weighted average of the covariances from the new location \( S_0 \) to all calibration points \( X_1, \ldots, X_n \) and the Lagrange multiplier. The square root of the kriging variance is called the kriging standard error.

In this case, the Kriging Standard Error (KSE) is 0.651. If it is assumed that the
errors are normally distributed, with 95 percent prediction interval using: Kriging Predictor ± 1.96*KSE. The value 1.96 comes from the standard normal distribution where 95 percent of the probability within the interval -1.96 to 1.96. This means that if predictions are made again and again from the same model, in the long run 95 percent of the time the prediction interval will contain the value at the prediction location. In this problem the prediction interval ranges from 24.67 to 27.59.

The proposed mean surface temperature (MST) prediction model derived by RK is:

\[ \hat{T}(s_0) = \sum_{k=0}^{7} \beta_k X_k(S_0) + \sum_{i=1}^{6} \lambda_i e(s_i) \hat{T}(s_0) \]

\[ = -42.804 + 0.107\text{LONG}(s_0) + 0.002\text{LAT}(s_0) - 0.124\text{MRH}(s_0) + 1.188\text{MRAD}(s_0) - 0.300\text{ELEV}(s_0) \]

\[ + 0.310\text{EVAP}(s_0) + (-0.333\text{DIST}(s_0)) - 0.924e(s_1) - 0.508e(s_2) \]

\[ - 0.038e(s_3) + 0.60e(s_4) + 0.238e(s_5) + 0.324e(s_6) (4.1) \]
4.7 Validation and Kriging Error Distribution

Using cross-validation, the study compared four error statistics summaries. To estimate how much of variation has been explained by Ordinary Kriging and regression kriging models, it shows that OK is not much worse than RK. Although RK is a better predictor, the difference is only about 12%. This is possibly because the variables elevation, evaporation and distance are also spatially continuous, and because the samples were equally spread in geographic space. Indeed Figures 4.4 and 4.5 again shows that the two maps do not differ much. Furthermore, the amount of variation explained by regression kriging is about 78%, which is satisfactory.

Table 4.10: Comparison of RK to MLR and OK

<table>
<thead>
<tr>
<th>Mod</th>
<th>MPE</th>
<th>RMSPE</th>
<th>AKSE</th>
<th>MSPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>OK</td>
<td>-0.014</td>
<td>1.06</td>
<td>2.50</td>
<td>0.221</td>
</tr>
<tr>
<td>RK</td>
<td>-0.017</td>
<td>1.02</td>
<td>2.06</td>
<td>0.203</td>
</tr>
<tr>
<td>MLR</td>
<td>1.067</td>
<td>1.83</td>
<td></td>
<td>0.238</td>
</tr>
</tbody>
</table>

4.8 Relationship between temperature values and the individual independent variables.

The Figures below present the relationship between Mean Surface Temperature and the independent variables (Mean Relative Humidity, Mean Radiation, Elevation, Evaporation and Distance) and as well a histogram plot and an autocorrelation function as further test for normality and autocorrelation. It shown that MST drops with MRH and DIST whereas MRAD, ELEV. and EVAP. have linear trend and scatter around the regression lines, which means that the residuals are significant.
Figure 4.6: Relationship between Mean Surface Temperature and Mean Relative Humidity

Figure 4.7: Relationship between Mean Surface Temperature and Mean Radiation

Figure 4.8: Relationship between Mean Surface Temperature and Evaporation
Figure 4.9: Relationship between Mean Surface Temperature and Elevation

Figure 4.10: Relationship between Mean Surface Temperature and Distance to nearby ocean

Figure 4.11: (i) Auto correlation function (acf) of MST (ii) A histogram of MST
4.9 Major Findings

This study was primarily concerned with modeling mean surface temperature of Nigeria as a function of elevation, radiation, humidity, distance to nearby sea, longitude, latitude and evaporation using geostatistical approach. The following were the findings:

i. Different semivariogram models were developed (Gaussian, spherical, linear and exponential) but the Gaussian and exponential models were therefore chosen as the most reliable models for interpolating MST and were used for the final map productions because the two models had the least residual sum of squares (RSS).

ii. All the independent variables (elevation, radiation, humidity, distance to nearby sea, longitude, latitude and evaporation) contributed significantly to the multiple linear regression, the ordinary kriging and the regression kriging models respectively.

iii. Cross validation of the three approaches (multiple linear regression, the ordinary kriging and the regression kriging) suggested the regression kriging model for long term prediction of mean surface temperature of Nigeria.
CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

This study developed the MST model using three approaches: Regression Kriging (RK), Ordinary Kriging (OK), Multiple Linear Regression (MLR) as a function mean relative humidity, mean radiation, elevation, evaporation, distance of state capitals of Nigeria to Lagos (ocean/sea), longitude and latitude (position of each of the state capitals) as predictors variables. The relationships between MST and some of the independent variables were significant at 0.05 levels of significance as indicated by the MLR. The experimental variogram of isotropic model for OK using Gaussian model, and the four directional variograms in the XYZ plane anisotropic using exponential model for the OK were fitted. The OK with zonal anisotropic in the Z direction showed that the spatial continuity in the directions of North-East and its vertical opposite South-west were stronger than in the directions of South-East and North-West. Also, it has shown that predictions of estimates and kriging variances for OK and RK were similar as shown in the maps. A cross validation of the three approaches were carried out and the Regression Kriging model was recommended for long term prediction of mean surface temperature of Nigeria.

5.2 Conclusion

This study developed MST models derived from Multiple Linear Regression (MLR) and two Geostatistical approaches: Ordinary Kriging (OK) and Regression Kriging (RK). Cross validations of the three approaches suggested the RK model for interpolation of long term Mean Surface Temperature of Nigeria.
5.3 Recommendations.

Further research is recommended to test whether other environmental factors, such as rainfall, wind speed and others might allow explanation of a larger proportion of the spatial variability displayed by surface temperature. As there are some additional geostatistical methods that were not evaluated in this study, it may be necessary to explore these methods to determine if they could generate a better representation of surface temperature across the study area.

5.4 Contribution to knowledge.

(1) From the works reviewed, the regression component (deterministic part) of the Regression Kriging was treated as a simple Linear regression while in our work, the deterministic component of the Regression Kriging was treated as Multiple Linear Regression.

(2) For Our Validation and Kriging Error Distribution, we used four Error Statistics unlike other works reviewed that drew their conclusion using one or two Error statistics.

(3) The result of the cross validation showed that the Regression Kriging (RK) has Mean Prediction Error (MPE) value of -0.017 while the Ordinary Kriging (OK) and the Multiple Linear Regression (MLR) have -0.014 and 1.067 respectively. A further comparison revealed that the (RK) has the least errors compared to the (OK) and the (MLR). We therefore recommend the Regression Kriging model as the best approach for prediction of mean surface temperature of Nigeria.
5.5 REFERENCES


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